# Simulation models and the Monte Carlo method

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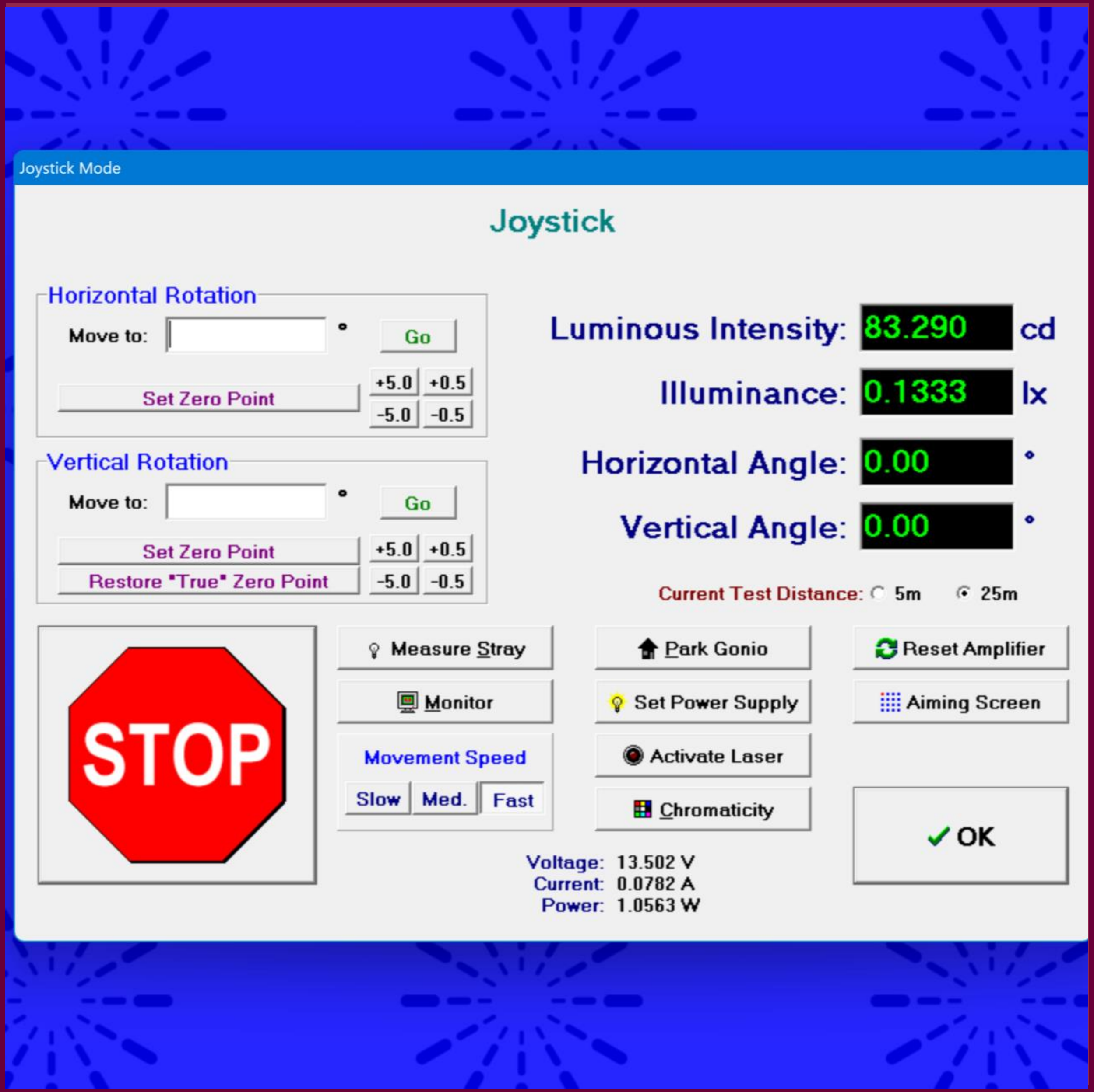
So you're taking some measurements and the readings are fluctuating…



# Which of these readings are correct?

- The average (mean)?
- The median value?
- The highest value?
- The lowest value?
- The most common value?
- None of the above?
- All of the above?

# My answer:

- All of the above!
- The fluctuations are there because the readings are samples from a **probability distribution**
- There is a “true” value (measurand), which is always unknown (and which may change over time)
- When we perform a measurement, we are making an estimate of the “true” value
- There are a lot of things that influence the measurement

What might influence the measurement?

- The probability distribution describes how these factors influence the measurements
- The nature of the probability distribution associated with a measurement is something that we call **measurement uncertainty**
- **So… every measurement is correct, because it is a sample, but none of them are the “true” value**

# Random vs Systematic

- We normally divide these types of effects into two categories: random and systematic
- Random:  fluctuations, usually around a mean value, may make some readings higher and some readings lower
- Systematic:  fixed effect which could make the reading higher or lower, but then stays the same and always influences the readings in the same way

What are examples of random and systematic effects?

# Statistical treatment

- “Type A” evaluation
  - For purely “random” variations
  - Able to be assessed by repeat measurements – take many measurements and average
  - The more measurements that we have, the more confident we are in the result

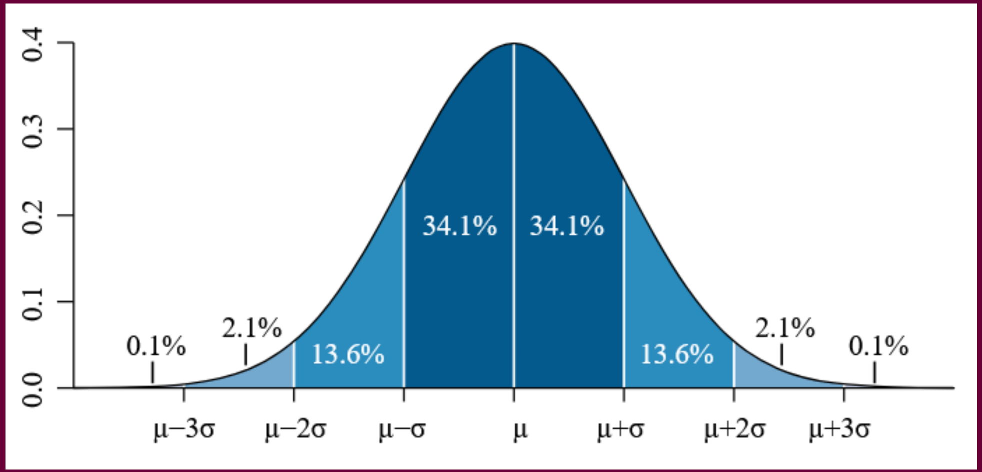
- “Type B” evaluation
  - For “unknown” effects, i.e. something other than repeated observations
  - Use other knowledge:
    - Calibration certificates;
    - Manufacturer’s specifications;
    - Scientific knowledge;
    - Previous experience;
    - Reference data from handbooks

# Probability distributions

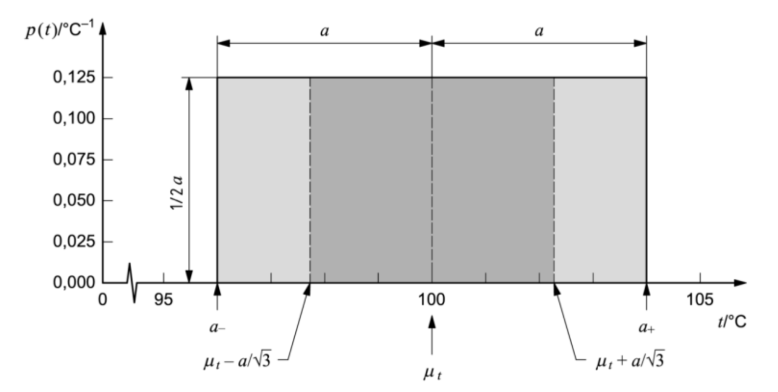
Source: Instrument Choice


- Normal distribution
  - Also called “Gaussian” distribution
  - For purely “random” variations
  - Able to be assessed by repeat measurements
  - Two standard deviations gives 95 % coverage

Source: Wikipedia




- Rectangular distribution
  - Only boundary conditions known
    - Display resolution
    - Manufacturer’s specifications
  - Equal probability between boundaries
  - Can’t be improved by repeat measurements

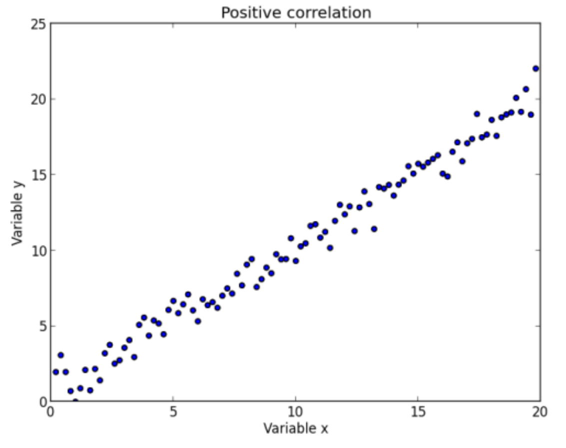



Source: JCGM 100:2008 Guide to the expression of uncertainty in measurement (“The GUM”)

Note: There are also other distributions, but these are the main ones we use…

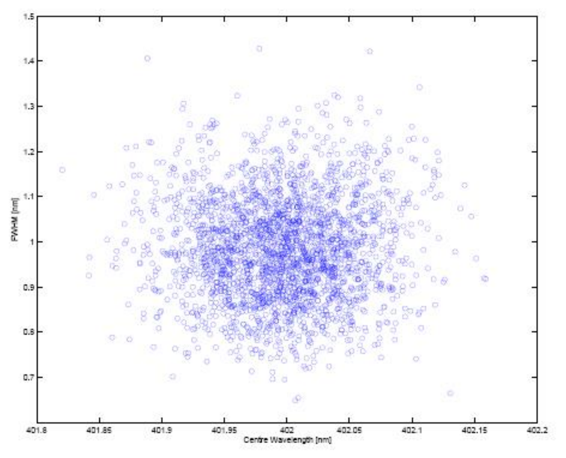
# Correlations

- Correlations are when one variable has an effect on another, or two different measurements are somehow linked
- Eg:
  - When measuring a spectrum, a wavelength error at one point will probably be similar to the next point
  - When monitoring a lamp output until it stabilises and the temperature or voltage is changing
- Normally, correlated terms are grouped together and assessed before combining with other variables
- Not covered here in detail




Strongly correlated

eg: intensity of a lamp and the applied voltage




Uncorrelated

eg: detector calibration and test distance measurement

# Now some examples!

- Excel examples for:
  - Normal distribution
  - Rectangular distribution

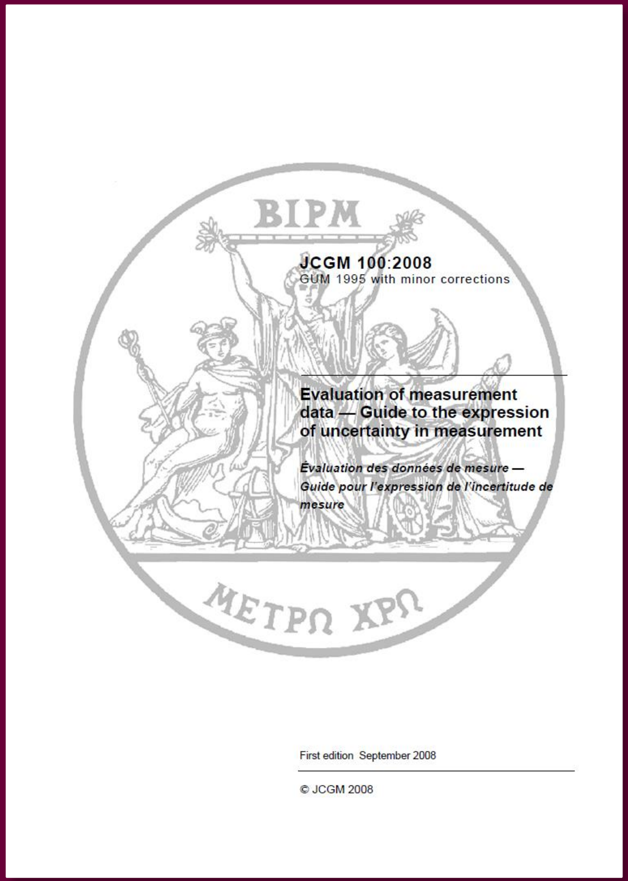
See Excel spreadsheet 01

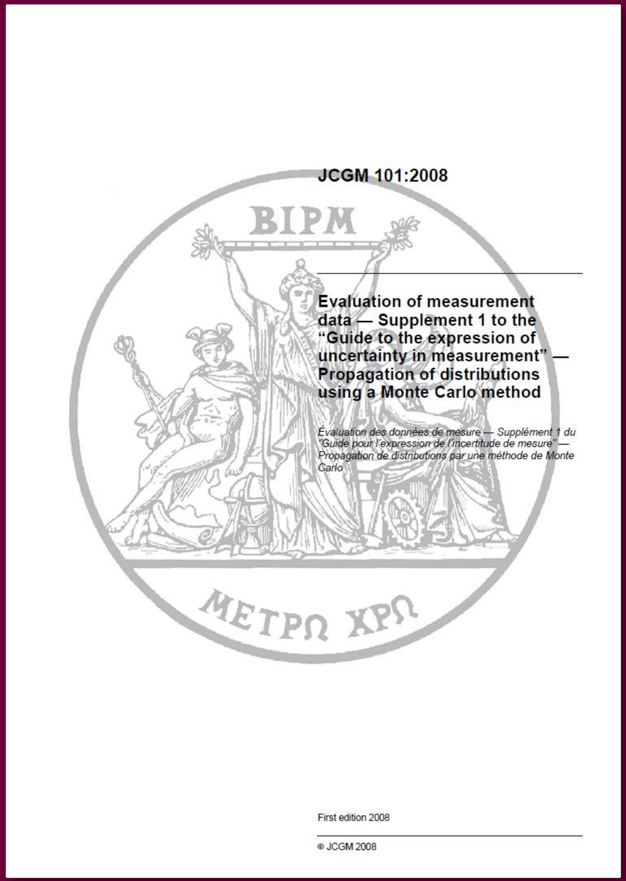
See Excel spreadsheet 02

- But then what happens when we mix these?

# And now the Monte Carlo method!

"The GUM"




These are freely downloadable from JCGM

# Definition

- The definition of Monte Carlo method, taken from JCGM 101 is:

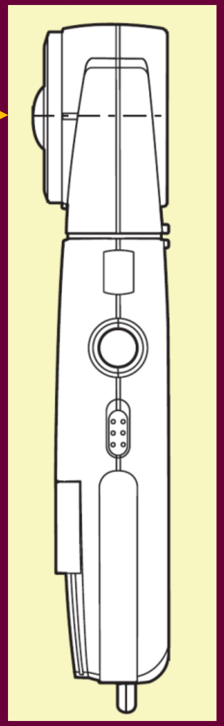
distribution of a scaled and shifted $t$-distribution in order to provide a coverage interval

**3.19**
**Monte Carlo method**
method for the propagation of distributions by performing random sampling from probability distributions

**3.20**

# Example 1

- Measurement of the luminous intensity of a lamp using the photometric inverse square law

$$I = E \times d^2$$

To start with, let's keep it simple!

# Example 1

- Say we have five measurements at different distances

| Measured distance (m) | Measured illuminance (lx) | Calculated lum. intensity (cd) |
|---|---|---|
| 1.000 | 213 | 213.00 |
| 1.500 | 94.9 | 213.53 |
| 2.000 | 53.4 | 213.60 |
| 2.500 | 34.2 | 213.75 |
| 3.000 | 23.7 | 213.30 |
| | Mean: | 213.44 |
| | Std Dev: | 0.26 |
| | 2SD/Avg: | 0.25% |

# Example 1

- Let's start with a simple uncertainty budget, thinking only of basic properties of the illuminance and distance measurements

| Uncertainty component | Value | Distribution |
|---|---|---|
| Detector calibration | 1 % (k = 2) | Normal (assumed) |
| Detector resolution | Variable | Rectangular |
| Distance meter calibration | 2 mm (k = 2) | Normal (assumed) |
| Distance meter resolution | 0.5 mm | Rectangular |

- The "k = 2" is a bit like saying that it is 2 standard deviations, i.e. 95 % coverage

See Excel spreadsheet 03

# Example 1

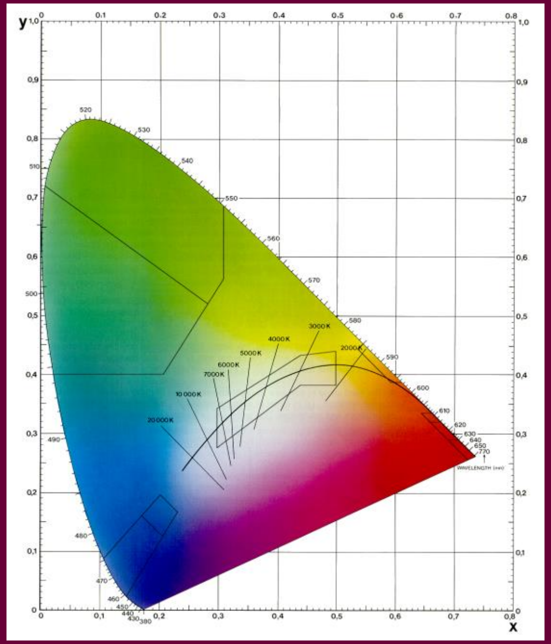
- Now let's add Stray Light into the mix
- Stray light will always add to the signal we are measuring
- To account for this, we would subtract it from the measured value
- In uncertainties, this is an asymmetrical component

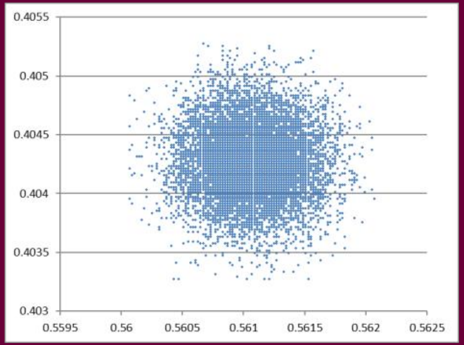
| Uncertainty component | Value | Distribution |
|---|---|---|
| Stray light | 2 % | Rectangular, asymmetrical |

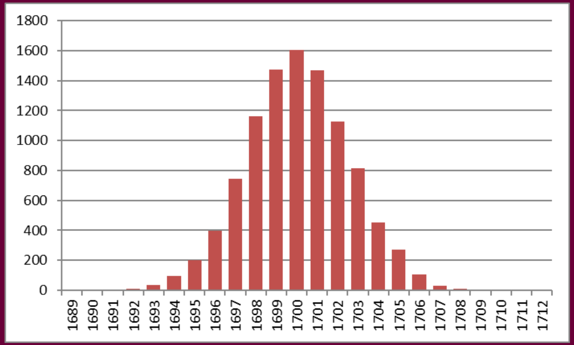
See Excel spreadsheet 04

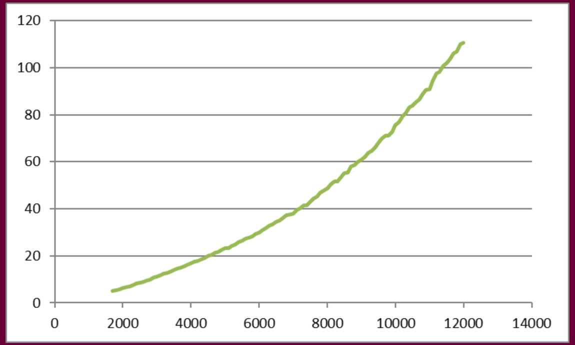
# Example 2

- How can we determine the Uncertainty of Correlated Colour Temperature (CCT)?
- This is a nonlinear parameter derived from chromaticity coordinates
- For a single CCT:
  - Generate 10,000 random iterations of uncertainty applied to a measured spectrum
  - For each sample, calculate the (x,y) coordinates
  - Calculate the CCT from these coordinates
  - Analyse the spread of the CCT statistically
- Repeat over the range of CCT values required



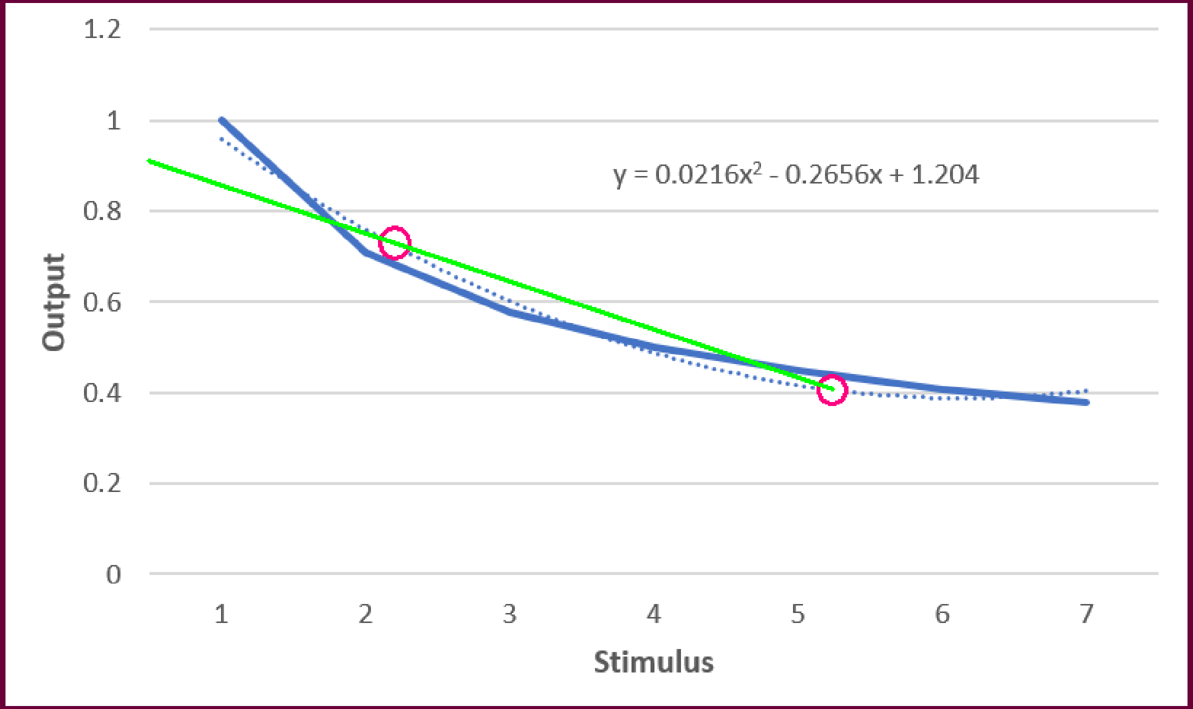


# Example 3

- Complex task involving polynomial curve fitting
- This is ideally suited to the Monte Carlo approach



# Further reading

- CIE 198:2011 contains much more detail on setting up models and evaluating measurement uncertainty
- It includes the Monte Carlo method and contains a number of (properly validated) spreadsheet examples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TECHNICAL REPORT

DETERMINATION OF MEASUREMENT UNCERTAINTIES IN PHOTOMETRY

CIE 198:2011

| | | | |
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# Conclusions

- Comparison of simulation models with experimental data can be evaluated in a similar way to measurement uncertainty
- One of the complicated aspects is the mixing of different probability distributions, particularly with asymmetry
- The Monte Carlo method uses repeat sampling of probability distributions for statistical evaluation of a measurement model
- Once the model is set up, it is straightforward to vary the input parameters to see the effect on the output

# Thank you for your kind attention

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